Improved lattice QCD with quarks: the 2 dimensional case

Jun-Qin Jiang[b], Xiang-Qian Luo[a,c,d], Zhong-Hao Mei[c,d], Hamza Jirari[e], Helmut Kröger[e], Chi-Min Wu[f]

[a] *CCAST(World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*

[b] *Department of Physics, Guangdong Institute of Education, Guangzhou 510303, China*

[c] *Department of Physics, Zhongshan University, Guangzhou 510275, China*[*]

[d] *Center for Computational Physics, School of Physics Science and Engineering, Zhongshan University, Guangzhou 510275, China*

[e] *Département de Physique, Université Laval, Québec, Québec G1K 7P4, Canada*

[f] *Institute for High Energy Physics, Academia Sinica, Beijing 100039, China*

(February 1, 2008)

Abstract

QCD in two dimensions is investigated using the improved fermionic lattice Hamiltonian proposed by Luo, Chen, Xu, and Jiang. We show that the improved theory leads to a significant reduction of the finite lattice spacing errors. The quark condensate and the mass of lightest quark and anti-quark bound state in the strong coupling phase (different from t'Hooft phase) are computed. We find agreement between our results and the analytical ones in the continuum.

PACS number(s): 11.15.Ha, 11.30.Qc, 11.30.Rd, 12.38.Gc

[*] Official and mailing Address. E-mail: stslxq@zsulink.zsu.edu.cn

I. INTRODUCTION

Wilson's lattice formulation of QCD is the most reliable and powerful non-perturbative approach to the physics of strongly interacting particles, but its progress was hampered by systematical errors mainly due to the finite value of the lattice spacing a.

Wilson's gluonic action differs from the continuum Yang-Mills action by order of $O(a^2)$, while the error of Wilson's quark action is bigger, i.e., the error being of the order $O(a)$. Due to quantum effects, there is an additional problem called "tadpole". If a and the bare coupling constant g were small enough, these errors would be negligible. Unfortunately, even in recent 4-dimensional lattice QCD simulations on the most powerful computers, the lattice spacing a is larger than $0.1 fm$ and the lattice coupling g_{lat} is bigger than 0.9. For these lattice parameters, violation of scaling is still obvious and the extrapolation of the results to the $a \to 0$ limit induces unknown systematic uncertainties when extracting continuum physics.

One of the most efficient ways for reducing these systematic errors is the Symanzik improvement program [1]: adding local, nearest neighbor or next-nearest-neighbor interaction terms to the Wilson lattice theory so that the finite a errors become higher order in a. During recent years, application of the Symanzik program has been a major topic. There have been several proposals on this subject:
(a) For the gauge sector, Lepage proposed a tadpole improved action [2], reducing the errors from $O(a^2)$ to $O(a^4)$. Luo, Guo, Kröger, and Schütte constructed a tadpole improved Hamiltonian [3,4] with the same accuracy.
(b) For the fermionic sector, Hamber and Wu (one author of the present paper) proposed the first improved action [5], by adding next-nearest-neighbor interaction terms to the Wilson quark action so that the $O(a)$ error is canceled. There have been some numerical simulations [6–8] of hadron spectroscopy using the Hamber-Wu action. Sheikholeslami and Wohlert [9], Lüscher and Weisz [10] worked on fermionic improvement and recent progress has been achieved on non-perturbative improvement via implementation of PCAC by the ALPHA collaboration [11]. Luo, Chen, Xu, and Jiang (two authors of the present paper) constructed an improved Hamiltonian [12], which was tested in the Schwinger model (QED_2). There are other possibilities for improving Wilson's quark theory [13]. The purpose of this work is to demonstrate numerically that fermionic improvement works also in the Hamiltonian formulation for the case of QCD in two dimensions.

Like Quantum Electrodynamics in 2 dimensions (QED_2) also Quantum Chromodynamics in 2 dimensions (QCD_2) has the properties of chiral symmetry breaking, an anomaly and confinement. However, the latter has gluonic self interactions, which makes it more similar to QCD in 4 dimensions (QCD_4) than to the Schwinger model. In addition to having confinement and chiral-symmetry breaking, QCD_2 has much richer spectrum because of the non-abelian gauge interactions between "gluons" and "quarks". Early in 1974, 't Hooft [14] did pioneering work on QCD_2 using the $1/N_C$ expansion (where QCD corresponds to the gauge symmetry group $\mathrm{SU(N_C)}$). He showed in the limit $N_C \to \infty$ that planar diagrams are dominant and they can be summed up in a bootstrap equation. From that he obtained a meson-like mass spectrum. This model has been extensively studied in the large N_C limit and also in the chiral limit $m_q \to 0$ (where m_q is the free quark mass). Zhitnitsky [16] has pointed out that there two distinct phases:

(i) $N_C \to \infty$ firstly, and then $m_q \to 0$ afterwards,
(ii) $m_q \to 0$, and then $N_C \to \infty$.
The first case corrersponds to $g << m_q$, which is the weak coupling regime. It descibes the phase considered by t'Hooft, where the following relation holds,

$$N_C \to \infty, \quad g^2 N_C = \text{const.} \quad m_q >> g \sim \frac{1}{\sqrt{N_C}}. \tag{1.1}$$

t'Hooft found a spectrum of states (numbered by n) given by

$$M_n \sim \pi^2 m_0 n, \quad m_0^2 = \frac{g^2 N_C}{\pi}. \tag{1.2}$$

Zhitnitsky [16] has computed the gluon condensate and the quark condensate. The quark condensate is given by

$$\langle \bar{\psi}\psi \rangle = -N_C \sqrt{\frac{g^2 N_C}{12\pi}}. \tag{1.3}$$

The second case corresponds to $g >> m_q$ which is the strong coupling regime. In the strong coupling regime the observed spectrum is different [17–19]. Steinhardt [19] has computed the masses of the following particles: soliton (baryon), anti-soliton (anti-baryon) and soliton-antisoliton (baryon-antibaryon) bound state. Bhattacharya [18] has shown that in the chiral limit there are free fields of mass

$$M = g\sqrt{\frac{N_C + 1}{2\pi}}. \tag{1.4}$$

Note that for $N_C = 1$ this coincides with the Boson mass in the Schwinger model. Grandou et al. [20] have obtained quark condensate in the chiral limit (for arbitrary g) and obtained

$$\langle \bar{\psi}\psi \rangle \sim -g N_C^{3/2} \tag{1.5}$$

which confirms Zhitnitsky's result obtained in the weak coupling regime.

Of course, QCD_2 is much simpler than QCD_4. It has been used to mimic the properties of QCD_4 such as vacuum structure, hadron scattering and decays, and charmonium picture. Unlike the massless Schwinger model, unfortunately, QCD_2 is no longer exactly solvable.

The first lattice study of QCD_2 was done by Hamer [21] using Kogut-Susskind fermions. He computed for SU(2) the mass spectrum in the t'Hooft phase. In 1991, Luo *et al.* [22] performed another lattice field theory study of this model (SU(2) and SU(3)) using Wilson fermions. As is shown later, the results for lattice QCD_2 with Wilson quarks were found to be strongly dependent on the unphysical Wilson parameter r, the coefficient of the $O(a)$ error term. The purpose of this paper is to show that in the case of QCD_2, the improved theory [12] can significantly reduce these errors.

The remaining part of the paper is organized as follows. In Sect. II, we review some features of the Hamiltonian approach as well as the improved Hamiltonian for quarks proposed in Ref. [12]. In Sect. III, the wave functions of the vacuum and the vector meson are constructed, and the relation between the continuum chiral condensate and lattice quark condensate is developed. The results for the quark condensate and the mass spectrum are presented in Sect. IV and discussions are presented in Sect. V.

II. IMPROVED HAMILTONIAN FOR QUARKS

Although numerical simulation in the Lagrangian formulation has become the main stream and a lot of progress has been made over the last two decades, there are areas where progress has been quite slow and new techniques should be explored: for example, computation of the S-matrix and cross sections, wave functions of vacuum, hadrons and glueballs, QCD at finite baryon density, or the computation of QCD structure functions in the region of small x_B and Q^2. In our opinion the Hamiltonian approach is a viable alternative [23,24] and some very interesting results [25–31] have recently been obtained. Many workers in the Lagrangian formulation nowadays have followed ideas similar to the Hamiltonian approach (where the time is continuos, i.e., $a_t = 0$) by considering anisotropic lattices with lattice spacings $a_t << a_s$. The purpose is to improve the signal to noise ratio in the spectrum computation [2].

In the last ten years, we have done a lot of work [22,32–37] on Hamiltonian lattice field theory, where the conventional Hamiltonian in the Wilson fermion sector is used:

$$
\begin{aligned}
H_f &= H_m + H_k + H_r, \\
H_m &= m \sum_x \bar{\psi}(x)\psi(x), \\
H_k &= \frac{1}{2a} \sum_{x,k} \bar{\psi}(x)\gamma_k U(x,k)\psi(x+k), \\
H_r &= \frac{r}{2a} \sum_{x,k} [\bar{\psi}(x)\psi(x) - \bar{\psi}(x)U(x,k)\psi(x+k)],
\end{aligned} \tag{2.1}
$$

where a is now the spacial lattice spacing a_s, $U(x,k)$ is the gauge link variable at site x in the direction $k = \pm j$ (j is the unit vector), and $\gamma_{-j} = -\gamma_j$, H_m, H_k, H_r are respectively the mass term, kinetic term and Wilson term. The Wilson term ($r \neq 0$), proportional to $O(ra)$, is introduced to avoid the fermion species doubling, with the price of explicit chiral-symmetry breaking even in the vanishing bare fermion mass limit. As discussed in Sec. I, the $O(a)$ error in H_f indeed leads to lattice artifacts if a or g_{lat} is not small enough.

Similar to the Hamber-Wu action [5], where some next-nearest-neighbor interaction terms are added to the Wilson action to cancel the $O(ra)$ error, we proposed a $O(a^2)$ improved Hamiltonian in Ref. [12]:

$$
\begin{aligned}
H_f^{improved} &= H_m + H_k^{improved} + H_r^{improved}, \\
H_k^{improved} &= \frac{b_1}{2a} \sum_{x,k} \bar{\psi}(x)\gamma_k U(x,k)\psi(x+k) \\
&+ \frac{b_2}{2a} \sum_{x,k} \bar{\psi}(x)\gamma_k U(x,2k)\psi(x+2k),
\end{aligned}
$$

$$H_r^{improved} = \frac{r}{2a}\sum_{x,k}\bar{\psi}(x)\psi(x)$$

$$-c_1\frac{r}{2a}\sum_{x,k}\bar{\psi}(x)U(x,k)\psi(x+k)$$

$$-c_2\frac{r}{2a}\sum_{x,k}\bar{\psi}(x)U(x,2k)\psi(x+2k). \tag{2.2}$$

Here $U(x,2k) = U(x,k)U(x+k,k)$ and the coefficients b_1, b_2, c_1 and c_2 are given by

$$b_1 = \frac{4}{3}, b_2 = -\frac{1}{6}, c_1 = \frac{4}{3}, c_2 = -\frac{1}{3}. \tag{2.3}$$

These coefficients are the same for any d+1 dimensions and gauge group. The results shown in this paper correspond to this set of parameters. However, the following set of paramters

$$b_1 = 1, b_2 = 0, c_1 = \frac{4}{3}, c_2 = -\frac{1}{3}, \tag{2.4}$$

where only the Wilson term is improved, gives very similar results. With the absence of the $O(ra)$ error, we expect that we can extract the continuum physics in a more reliable way.

Lattice QCD_2 in the Hamiltonian approach has some nice features which simplify the computations considerably:

(a) The magnetic interactions are absent and the tadpole factor $U_0 = 1$ due to the fact that $U_p = U_p^\dagger = 1$. Therefore, there is only color-electric energy term in the gluonic Hamiltonian:

$$H_g = \frac{g^2}{2a}\sum_{x,j}E_j^\alpha(x)E_j^\alpha(x), \tag{2.5}$$

with $j = \vec{1}$ and $\alpha = 1, ..., N_c^2 - 1$.

(b) The quantum (tadpole) effects, if any, are highly suppressed in the fermionic sector as $O(g_{lat}^2 a) \to O(a^3)$ and in the gluonic sector as $O(g_{lat}^2 a^2) \to O(a^4)$. The reason is the super-renormalizability of 1+1 dimensional theories, where

$$g_{lat} = ga. \tag{2.6}$$

All this means that in 1+1 dimensions, classical improvement of the fermionic Hamiltonian is sufficient. In Ref. [12], the mass spectrum of the Schwinger model was used to test the improved program. In the following sections, we will provide evidence in QCD_2 to support the efficiency of the improved Hamiltonian. The results for the quark condensate and mass spectrum further confirm our expectation.

III. VACUUM AND VECTOR PARTICLE STATES

The vacuum wave function is constructed in the same way as in [12]:

$$|\Omega\rangle = exp(iS)|0\rangle, \tag{3.1}$$

where

$$S = \sum_{n=1}^{N_{trun}^S} \theta_n S_n$$

$$S_1 = i\sum_{x,k} \psi^\dagger(x)\gamma_k U(x,k)\psi(x+k)$$

$$S_2 = i\sum_{x,k} \psi^\dagger(x)\gamma_k U(x,2k)\psi(x+2k)$$

$$S_3 = i\sum_{x,k} \psi^\dagger(x)\gamma_k U(x,3k)\psi(x+3k)$$

$$... \tag{3.2}$$

with θ_n determined by minimizing the vacuum energy. $|0\rangle$ is the bare vacuum defined by

$$\xi(x)|0\rangle = \eta(x)|0\rangle = E_j^\alpha(x)|0\rangle = 0. \tag{3.3}$$

Here ξ and $\eta^\dagger$ are respectively the up and down components of the ψ field. Such a form of the fermionic vacuum (3.2) has also been discussed extensively in the literature [22,32–37].

One is interested in computing the wave function of the lowest lying energy state which is the flavor-singlet vector meson $|V\rangle$, similar to the case of the Schwinger model. The wave function is created by a superposition of some operators V_n with the given quantum numbers [12,38]

$$V_0 = i\sum_{x} \bar{\psi}(x)\gamma_1\psi(x),$$

$$V_1 = i\sum_{x,k} \bar{\psi}(x)\gamma_1 U(x,k)\psi(x+k),$$

$$V_2 = i\sum_{x,k} \bar{\psi}(x)\gamma_1 U(x,2k)\psi(x+2k),$$

$$V_3 = i\sum_{x,k} \bar{\psi}(x)\gamma_1 U(x,3k)\psi(x+3k),$$

$$\ldots \tag{3.4}$$

acting on the vacuum state $|\Omega\rangle$, i.e.,

$$|V\rangle = \sum_{n=0}^{N^V_{trun}} A_n[V_n - \langle\Omega|V_n|\Omega\rangle]|\Omega\rangle. \tag{3.5}$$

The criterion for choosing the truncation orders N^S_{trun} and N^V_{trun} is the convergence of the results. An estimate for the vector mass M_V is the lowest eigenvalue $min(E_V)$ of the following equations

$$\sum_{n_1=0}^{N^V_{trun}} (H^V_{n_2n_1} - E_V U^V_{n_2n_1})A_{n_1} = 0,$$

$$det|H^V - E_V U^V| = 0, \tag{3.6}$$

where the coefficients A_{n_1} are determined by solving the equations, and the matrix elements $H^V_{n_2n_1}$ and $U^V_{n_2n_1}$ are defined by

$$H^V_{n_2n_1} = \langle V_{n_2}|[H_f^{improved} + H_g]|V_{n_1}\rangle^L,$$

$$U^V_{n_2n_1} = \langle V_{n_2}|V_{n_1}\rangle^L. \tag{3.7}$$

Here the superscript L means only the matrix elements proportional to the lattice size L are retained (those proportional to higher order of L do not contribute). Detailed discussions can be found in [38]. We can estimate the continuum M_V from

$$\frac{M_V}{g} = \frac{aM_V}{g_{lat}}, \tag{3.8}$$

if the right hand side is independent of g_{lat}.

In lattice field theory with Wilson fermions, chiral symmetry is explicitly broken even in the bare vanishing mass limit. Therefore, the fermion condensate $\langle\bar{\psi}\psi\rangle_{free}$ is non-vanishing and should be subtracted in a way described in Refs. [22,35,36,39]:

$$\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{free}, \tag{3.9}$$

where for one flavor

$$\langle\bar{\psi}\psi\rangle = \frac{1}{LN_c}\langle\Omega|\bar{\psi}\psi|\Omega\rangle_{m=0}$$

$$\langle\bar{\psi}\psi\rangle_{free} = \frac{1}{LN_c}\frac{\partial E_{\Omega_{free}}}{\partial m}|_{m=0}$$

$$= -\int_{-\pi/a}^{\pi/a} dpa\{[\frac{r}{a}(1 - c_1 cospa - c_2 cos2pa)]^2$$

$$+[\frac{sinpa}{a}(b_1+b_2cospa)]^2\}$$

$$/\{[\frac{r}{a}(1-c_1cospa-c_2cos2pa)]^2$$

$$+[\frac{sinpa}{a}(b_1+b_2cospa)]^2\}^{1/2}. \tag{3.10}$$

From $\langle\bar{\psi}\psi\rangle_{sub}$, we can get an estimate of the continuum quark condensate $\langle\bar{\psi}\psi\rangle_{cont}$

$$\frac{\langle\bar{\psi}\psi\rangle_{cont}}{g}=\frac{\langle\bar{\psi}\psi\rangle_{sub}}{g_{lat}}, \tag{3.11}$$

if the right hand side does not depend on the bare coupling g_{lat}.

It is well known that spontaneous chiral-symmetry breaking originates from the axial anomaly and there is no Goldstone pion in quantum field theory in 1+1 dimensions. Therefore, for Wilson fermions, one can not fine-tune the (r,m) parameter space as in QCD_4 to reach the chiral limit. However, the chiral limit can be approximated as the $m\rightarrow 0$ limit as long as the lattice spacing error is sufficiently small. For Wilson fermions, as discussed in Ref. [40], this is not well justified for finite g_{lat} and a because of the $O(ra)$ error, but for the improved theory, as shown in Refs. [7,12], this would be a reasonably good approximation since the chiral-symmetry breaking term is much smaller, i.e., $O(ra^2)$.

IV. RESULTS FOR QUARK CONDENSATE AND VECTOR MASS

To increase the accuracy of the techniques described in Sect. III, we include higher order contributions in Eq. (3.2) than those in Ref. [12] so that better convergence of the results can be obtained.

Figure 1 and Figure 3 show $\langle\bar{\psi}\psi\rangle_{sub}/g_{lat}$ as a function of $1/g_{lat}^2$ in 2-dimensional lattice SU(2) and SU(3), respectively, gauge theories with Wilson fermions. Figure 5 and Figure 7 show aM_V/g_{lat} as a function of $1/g_{lat}^2$ in 2-dimensional lattice SU(2) and SU(3), respectively, gauge theories with Wilson fermions. As one can see, the results for $r=1$ deviate obviously from those for $r=0.1$, which is attributed to the $O(ra)$ error of the Wilson term.

The corresponding results from the Hamiltonian with improvement are presented in Figure 2, Figure 4, Figure 6, and Figure 8. One observes that the differences between the results for $r=1$ and $r=0$ are significantly reduced. Most impressively, the data for the quark condensate coincide each other. A similar r test has also been used in [15] for checking the efficiency of the improvement program.

To get an idea about how QCD_2 behaves for large N_C, we have computed the quark condensate $\langle\bar{\psi}\psi\rangle$ in the chiral limit for $N_C=3,4,5,6$. This is shown in Figure 9. We have compared our numerical results with the theoretical result by Zhitnitsky [16], Eq.(1.3). Although Zhitnitsky's result was obtained in the weak coupling phase, it qualitatively agrees with the strong coupling result of Ref. [20]. Remarkably, our results agree very well with Zhitnitsky's weak coupling result. Secondly, we show in Figure 10 the lowest lying mass of the mass spectrum again for $N_C=3,4,5,6$ in the chiral limit. Note that this particle

corresponds to the vector particle in the Schwinger model. In QCD_2 it corresponds to free particles (the mass of which remains finite when m_q goes to zero) distinct from "sine-Gordon" soliton particles (the mass of which goes to zero when m_q goes to zero, see [19]). We compare our numerical result with the analytical strong coupling result by Bhattacharya [18], Eq.(1.4). Again we find agreement.

V. CONCLUSIONS

In this work we have shown that fermionic improvement works also in the Hamiltonian lattice formulation for the case of QCD_{1+1}. We have computed the quark condensate and the flavor singlet vector mass using the $O(a^2)$ improved Hamiltonian for quarks [12] proposed by Luo, Chen, Xu, and Jiang. In comparison with the results corresponding to the Wilson fermions without improvement, we indeed observe significant reduction of the finite lattice spacing error $O(ra)$. By comparison with analytical results for the quark condensate and the vector mass we find good agreement. In our opinion, this is the first lattice study of QCD_{1+1} which gives results in the strong coupling phase (in contrast to the t'Hooft phase). In particular we present results for different gauge groups ($N_C = 2, 3, 4, 5, 6$). We believe that the application of the Symanzik improvement program to QCD in 3+1 dimensions will be very promising.

ACKNOWLEDGMENTS

We are grateful to Q. Chen, S. Guo, J. Li and J. Liu for useful discussions. X.Q.L. is supported by the National Natural Science Fund for Distinguished Young Scholars, supplemented by the National Natural Science Foundation of China, fund for international cooperation and exchange, the Ministry of Education, and the Hong Kong Foundation of the Zhongshan University Advanced Research Center. H.K. is grateful for support by NSERC Canada.

REFERENCES

[1] K. Symanzik, Nucl. Phys. B **226**, 187 (1983).

[2] P. Lepage, "Redesigning Lattice QCD", in Proc. of Schladming Winter School 1996, hep-lat/9607076.

[3] X.Q. Luo, S.H. Guo, H. Kröger, and D. Schütte, Nucl. Phys. B(Proc. Suppl.) **63A-C**, 931 (1998).

[4] X.Q. Luo, S.H. Guo, H. Kröger, and D. Schütte, Phys. Rev. D, in press.

[5] H.W. Hamber and C.M. Wu, Phys. Lett. B **133**, 251 (1983).

[6] M.P. Lombardo, G. Parisi and A. Vladikas, Nucl. Phys. B **395**, 388 (1993).

[7] A. Borici and P. de Forcrand, Nucl. Phys. B(Proc. Suppl.)**53**, 873 (1997).

[8] H. Fiebig and R. Woloshyn, Nucl. Phys. B(Proc. Suppl.)**53**, 886 (1997).

[9] S. Sheikholeslami and R.Wohlert, Nucl. Phys. B **259**, 572 (1985).

[10] M. Lüscher and P. Weisz, Phys. Lett. B **158**, 250 (1985).

[11] M. Lüscher, Advanced Lattice QCD, hep-lat/9802029.

[12] X.Q. Luo, Q.Z. Chen, G.C. Xu and J.Q. Jiang, Phys. Rev. D **50**, 501 (1994).

[13] X.Q. Luo, Computer Phys. Commun. **94**, 119 (1996).

[14] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974).

[15] M. Alford, T. Klassen, G.P. Lepage, Nucl. Phys. B(Proc. Suppl.)**53**, 861 (1997).

[16] A.R. Zhitnitsky, Phys. Lett. B **165**, 405 (1985).

[17] V. Baluni, Phys. Lett. B **90**, 407 (1990).

[18] G. Bhattacharya, Nucl. Phys. B **205**[FS5], 461 (1982).

[19] P.J. Steinhardt, Nucl. Phys. B **176**, 100 (1980).

[20] T. Grandou, H.T.Cho and H.M. Fried, Phys. Rev. D **37**, 946 (1988).

[21] C.J. Hamer, Nucl. Phys. B **195**, 503 (1980).

[22] X.Q. Luo, B.P. He, Q.Z. Chen, and S.H. Guo, Z. Phys. C **51**, 423 (1991).

[23] H. Kröger, Phys. Repts. **210**, 45 (1992).

[24] S.H. Guo and X.Q. Luo, "Recent Advances in Hamiltonian Lattice Gauge Theory", in Experimental and Theoretical Study of Glueballs, CCAST-WL Workshop Series: **76** 127 (1997).

[25] X.Q. Luo and Q.Z. Chen, Mod. Phys. Lett. A **11**, 2435 (1996).

[26] X.Q. Luo, Q.Z. Chen, S.H. Guo, X.Y. Fang, and J.M. Liu, Nucl. Phys. B(Proc. Suppl) **53**, 243 (1997).

[27] X.Q. Luo, Q.Z. Chen, P.F. Cai, and S.H. Guo, Commun. Theor. Phys. **29**, 595 (1998).

[28] J. Brière, H. Kröger, Phys. Rev. Lett. **63**, 848 (1989).

[29] H. Kröger and N. Scheu, Phys. Rev. D **56**, 1455 (1997).

[30] H. Kröger and N. Scheu, Phys. Lett. B **429**, 58 (1998).

[31] D. Schütte, W.H. Zheng, C.J. Hamer, Phys. Rev. D **55**, 2974 (1997).

[32] X.Q. Luo, Q.Z. Chen, and S.H. Guo, High Energy Phys. and Nucl. Phys. (Allerton, New York) **13**, 157 (1989).

[33] X.Q. Luo, Q.Z. Chen, and S.H. Guo, Z. Phys. C **47**, 635 (1990).

[34] X.Q. Luo and Q.Z. Chen, J. Phys. G **16**, 1181 (1990).

[35] Q.Z. Chen and X.Q. Luo, Phys. Rev. D **42**, 1293 (1990).

[36] X.Q. Luo, Commun. Theor. Phys. **16**, 505 (1991).

[37] X.Q. Luo and Q.Z. Chen, Phys. Rev. D **46**, 814 (1992).

[38] X.Y. Fang, X.Q. Luo, G.C. Xu, and Q.Z. Chen, Z. Phys. C **54**, 587 (1992).

[39] X.Q. Luo, Z. Phys. C **48**, 283 (1990).
[40] V. Azcoiti, G. DiCarlo, A. Galante, A. Grillo, V. Laliena, Phys. Rev. D **53**, 5069 (1996).

FIGURES

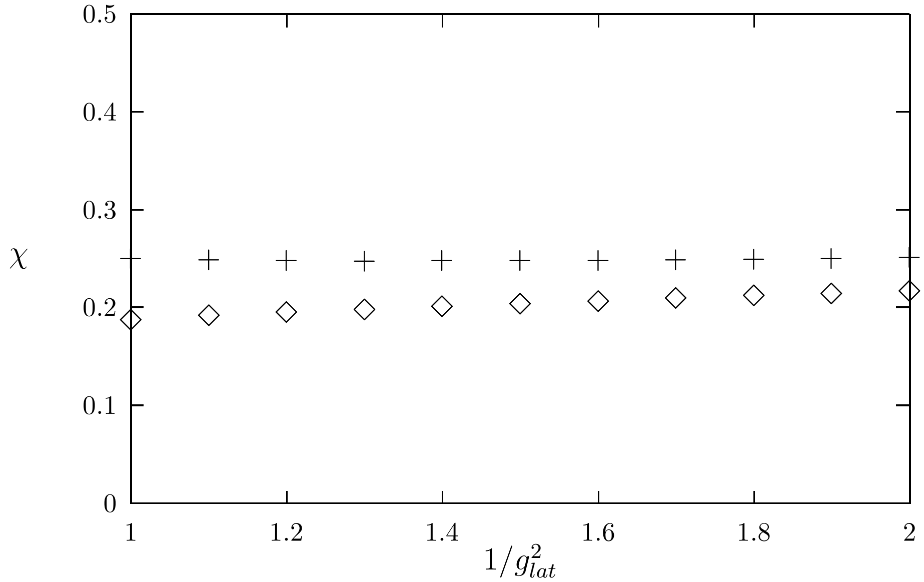

FIG. 1. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(g_{lat}N_c)$ versus $1/g^2_{lat}$ for $N_C = 2$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

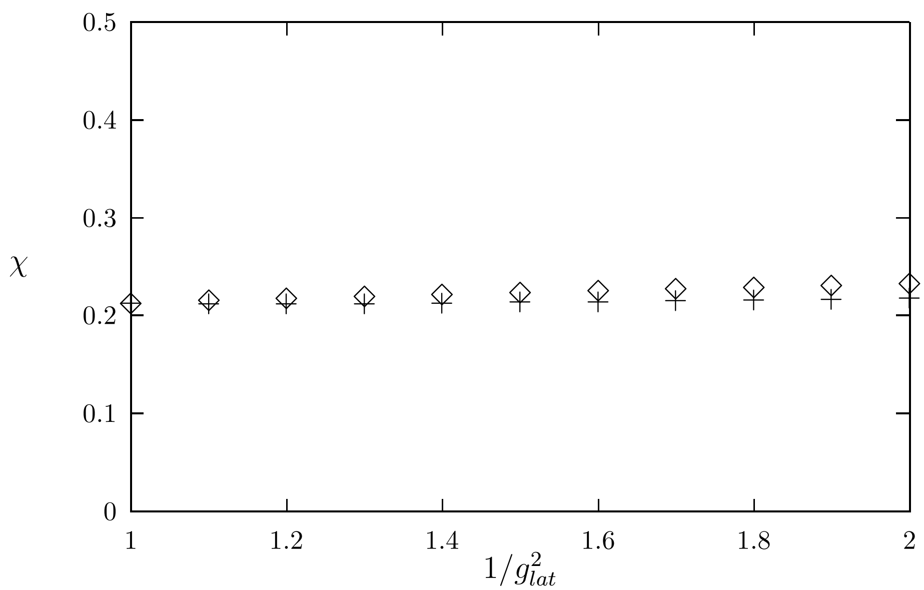

FIG. 2. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(g_{lat}N_c)$ versus $1/g^2_{lat}$ for $N_C = 2$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

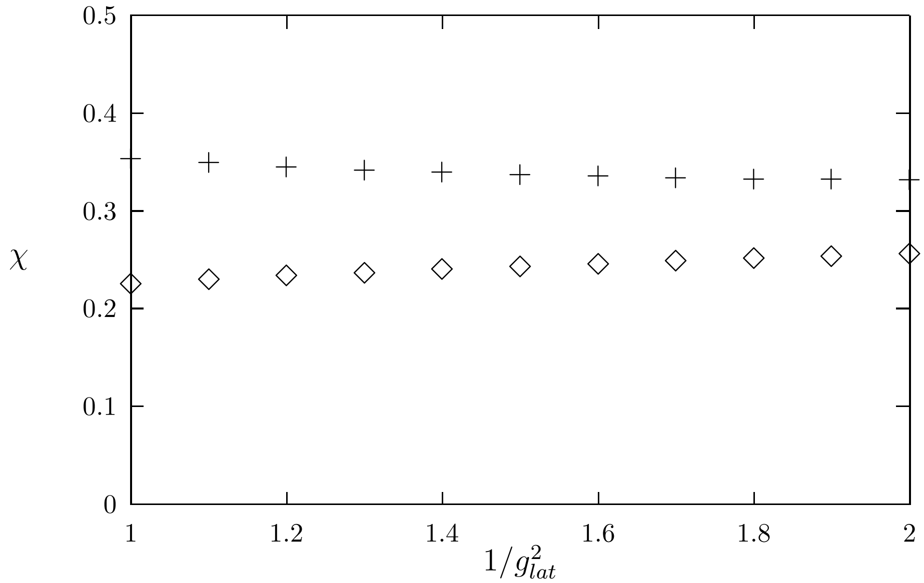

FIG. 3. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(g_{lat}N_c)$ versus $1/g_{lat}^2$ for $N_C = 3$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

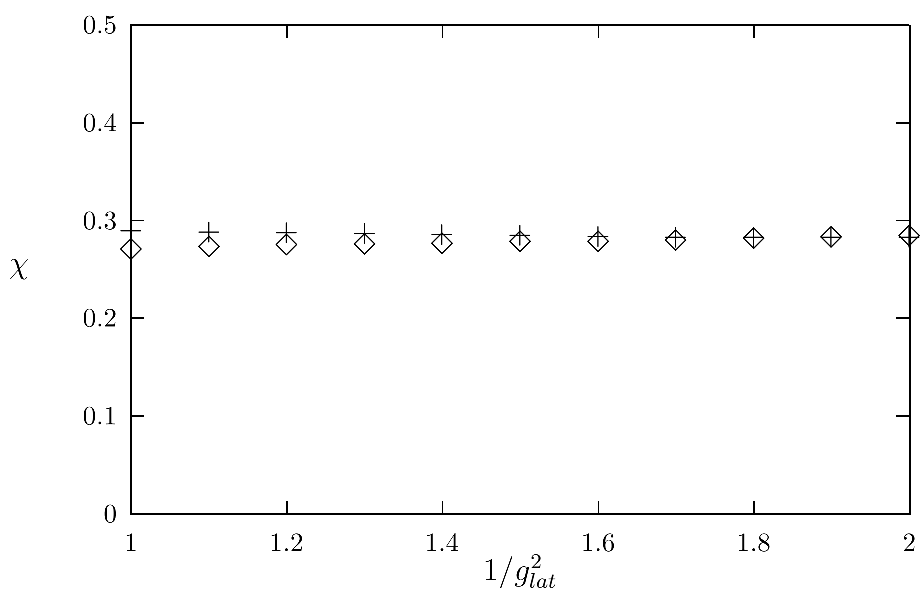

FIG. 4. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(g_{lat}N_c)$ versus $1/g_{lat}^2$ for $N_C = 3$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

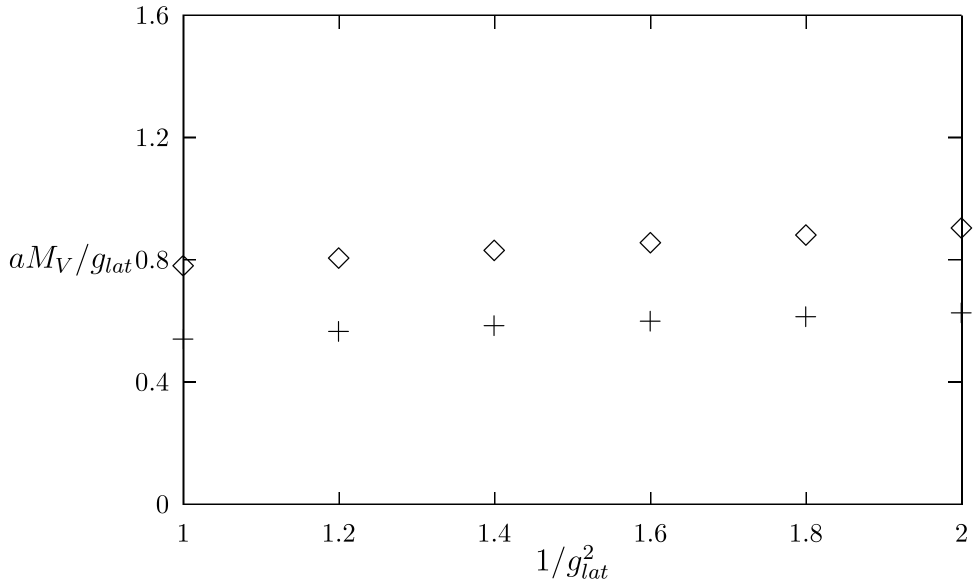

FIG. 5. aM_V/g_{lat} versus $1/g_{lat}^2$ for $N_C = 2$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

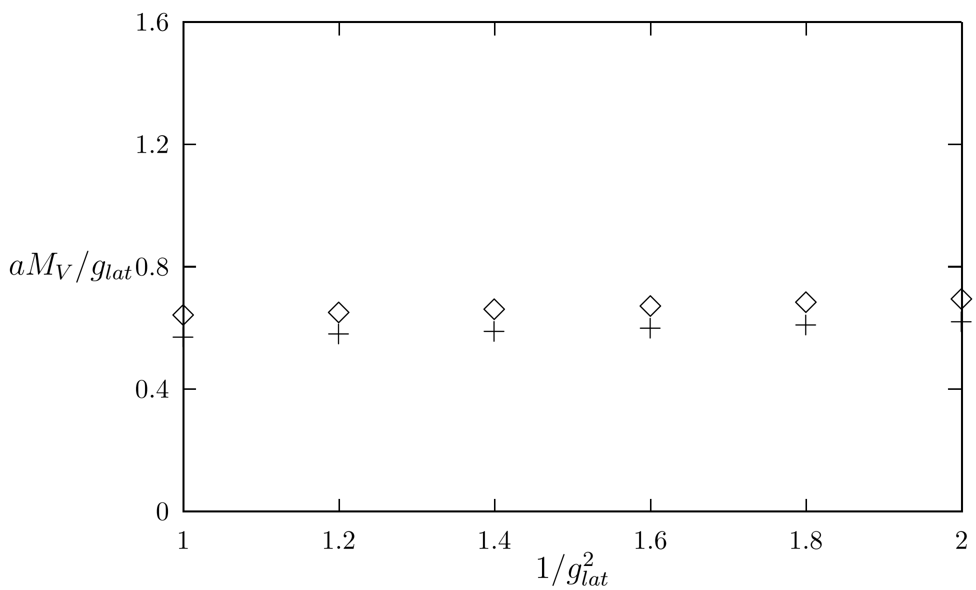

FIG. 6. aM_V/g_{lat} versus $1/g_{lat}^2$ for $N_C = 2$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

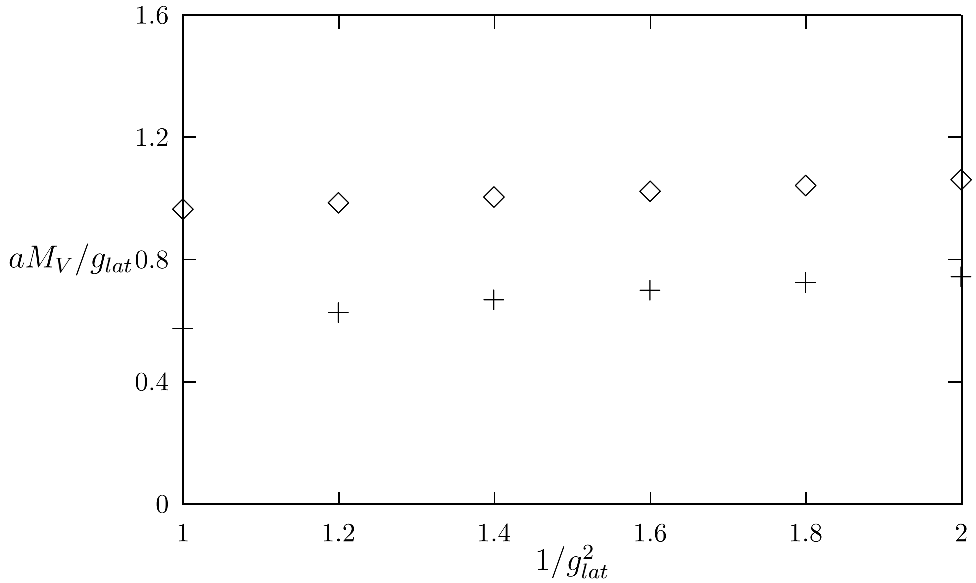

FIG. 7. aM_V/g_{lat} versus $1/g^2_{lat}$ for $N_C = 3$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

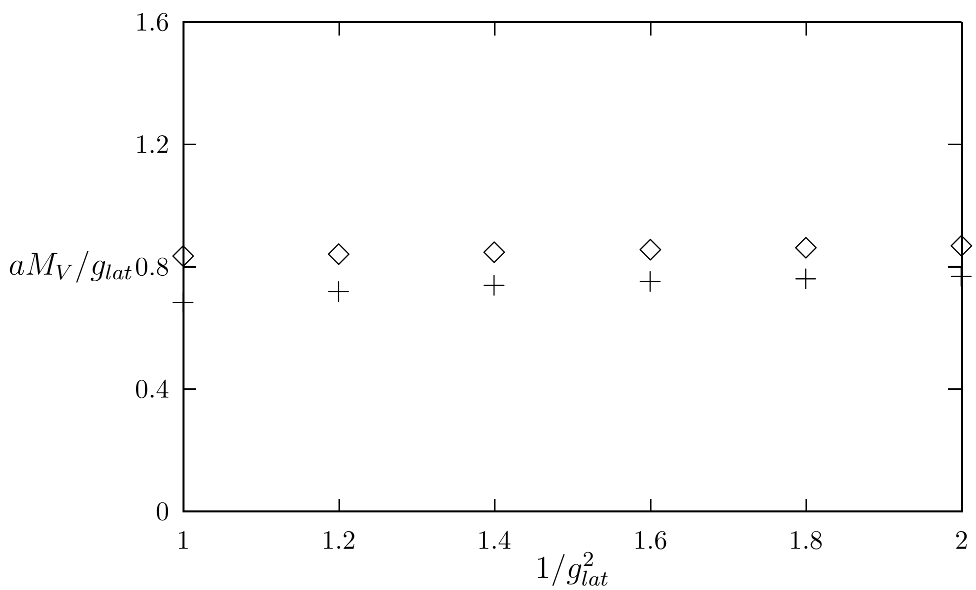

FIG. 8. aM_V/g_{lat} versus $1/g^2_{lat}$ for $N_C = 3$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

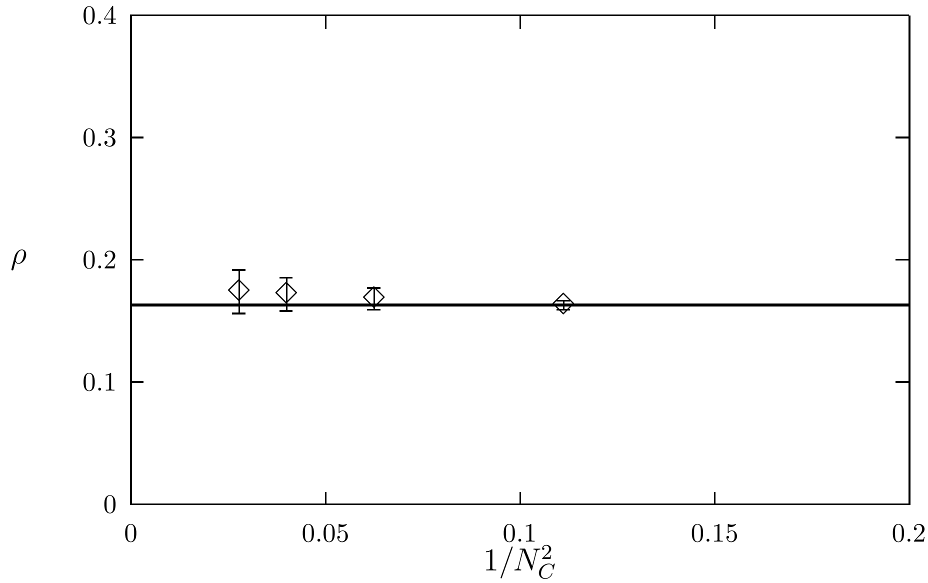

FIG. 9. $\rho = -\langle\bar{\psi}\psi\rangle_{cont}/(gN_C^{3/2})$ in the continuum versus $1/N_C^2$. The error bars are estimated from the data for different r. The full line gives the analytical result from Ref. [16].

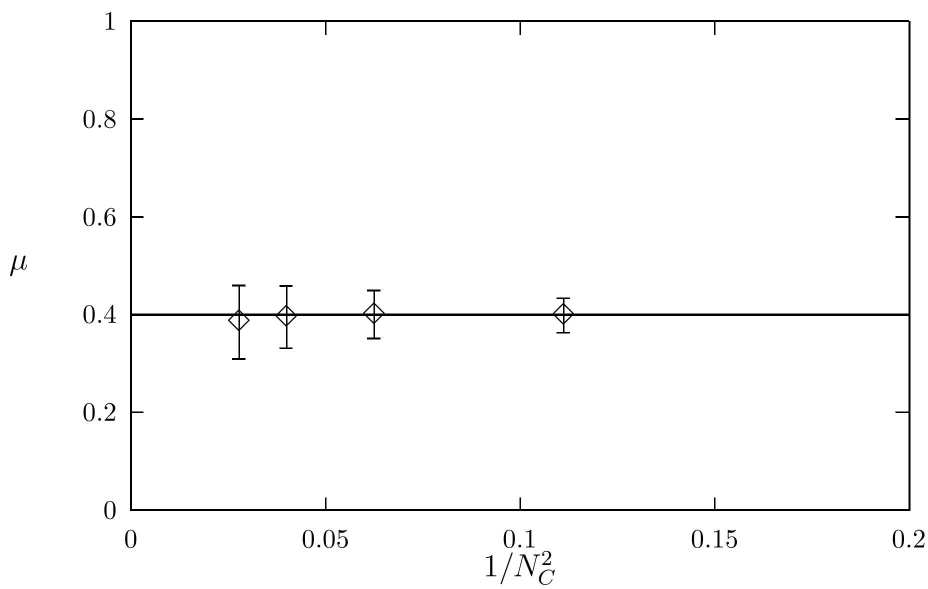

FIG. 10. $\mu = M_V/(g\sqrt{N_C+1})$ in the continuum versus $1/N_C^2$. The error bars are estimated from the data for different r. The full line gives the analytical result from Ref. [18].